Eur. Phys. J. B **1**, 269–272 (1998)

THE EUROPEAN PHYSICAL JOURNAL B

Rapid Note

Dynamics of selfavoiding tethered membranes. I. Model A dynamics (Rouse model)

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Received: 5 September 1997 / Accepted: 17 November 1997

Abstract. The dynamical scaling properties of selfavoiding polymerized membranes with internal dimension D are studied using model A dynamics. It is shown that the theory is renormalizable to all orders in perturbation theory and that the dynamical scaling exponent z is given by $z = 2 + D/\nu^*$. This result applies especially to membranes ($D = 2$) but also to polymers ($D = 1$).

PACS. 05.70.Fh Phase transitions: general aspects – 11.10.Gh Renormalization – 11.25.-w Theory of fundamental strings

Polymers and polymerized flexible membranes show interesting statistical properties. Polymers have been investigated since long time, both static and dynamic [1]. Selfavoiding polymerized membranes have attracted a remarkable interest during the last years. Their static properties have been studied numerically [2–6], experimentally [7,8], and analytically [9–13]. Dynamics has been regarded using scaling arguments for polymers [14,15] and membranes [16]. For polymers, a renormalization group analysis has been performed at 1- [17–22] and 2-loop [23] order.

For membranes, the analytical approach, inspired from polymer theory [1], relies on renormalization group and ε-expansion methods. It was initiated in [9,10], where it was used to perform calculations at 1-loop order. Its consistency to all orders in perturbation theory has been established in [11,24]. Recently, 2-loop calculations have been performed, which give reliable results for all embedding dimensions [12,13].

In this letter we address the question of the dynamics of such membranes, always including polymers as a special case.

The membrane is modeled by a continuum model à la Edwards: the embedding of the D-dimensional membrane in d-dimensional bulk space is described by the mapping $x \in \mathbb{R}^D \to r(x) \in \mathbb{R}^d$. The Hamiltonian, which describes the static properties of the membrane, is

$$\mathcal{H}[r] = \frac{1}{2}\int_x \big(\nabla r(x)\big)^2 + b\int_x\int_y \delta^d\big(r(x) - r(y)\big), \qquad (1)$$

where b is the coupling constant. The dynamics of the membrane is given by the Langevin-equation

$$\dot{r}(x,t) = -\lambda\frac{\delta H}{\delta r(x,t)} + \zeta(x,t) \qquad (2)$$

which models the purely diffusive motion of the membrane (model A in the terminology of [25]). The Gaussian noise $\zeta(x,t)$ has the correlation

$$\overline{\zeta(x,t)\zeta(x',t')} = 2\lambda\delta^D(x-x')\delta(t-t'). \qquad (3)$$

This Langevin equation can be formulated using an effective field theory [26] with action

$$J[r,\tilde{r}] = \int_x\int_t \tilde{r}(x,t)\left(\dot{r}(x,t) + \lambda\frac{\delta H}{\delta r(x,t)}\right) - \lambda\tilde{r}(x,t)^2. \qquad (4)$$

Expectation values are calculated by integrating over all fields r and $\tilde{r}$. Prepoint-discretization is used and the average over the noise has been taken. (This generated the term quadratic in $\tilde{r}$.)

Perturbation theory is performed by expanding about the Gaussian theory. We use the free propagator G and

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correlator C in position-space

$$C(x,t) := \frac{1}{d}\left\langle \frac{1}{2}(r(x,t)-r(0,0))^2 \right\rangle_0 \tag{5}$$
$$= |x|^{2\nu}\, \Gamma^{-1}(-\nu) \times\left(-\frac{1}{\nu}\left(\frac{4\lambda|t|}{x^2}\right)^{\nu} + \int_0^{\frac{x^2}{4\lambda|t|}} \frac{\mathrm{d}s}{s} s^{-\nu}\left(\mathrm{e}^{-s}-1\right)\right)$$
$$G(x,t) := \frac{1}{d}\langle r(x,t)\tilde{r}(0,0)\rangle_0$$
$$= \Theta(t)(4\pi\lambda|t|)^{-D/2}\,\mathrm{e}^{-x^2/4\lambda|t|}\, S_D(2-D), \tag{6}$$

where S_D is the volume of the unit sphere in D dimensions,

$$S_D = \frac{2\pi^{D/2}}{\Gamma(D/2)} \tag{7}$$

and

$$\nu = \frac{2-D}{2}. \tag{8}$$

Some normalization-factors have been absorbed into the measure in order to have

$$C(x,t) \approx |x|^{2-D} \quad \text{for} \quad x^2 \gg \lambda|t|. \tag{9}$$

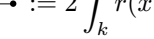
(For details compare appendix A of [13].) If x^2 is much smaller than $\lambda|t|$, the correlator approaches the finite value

$$C(x,t) = \frac{(4|t|\lambda)^{\nu}}{\Gamma(D/2)} + \mathcal{O}(x^2). \tag{10}$$

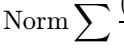
We should mention that the propagator is simply related to the time-derivative of the correlator

$$G(x,t) = \Theta(t)\frac{1}{\lambda}\dot{C}(x,t). \tag{11}$$

We are now in a position to construct the perturbation-theory. The interaction vertex is:


$$:= 2\int_k \tilde{r}(x,t)(ik)\,\mathrm{e}^{ik(r(x,t)-r(y,t))}. \tag{12}$$

The perturbative expansion of an observable $\mathcal{O}$ can be written as

$$\langle\mathcal{O}\rangle = \mathrm{Norm}\sum_n \frac{(\lambda b)^n}{n!}\int \langle \mathcal{O}$$

$$^{n}\rangle^{c}, \tag{13}$$

where the normalization Norm has to be chosen so that $\langle 1\rangle = 1$ and the integral is taken over all arguments of the interaction vertex. We claim that divergences only occur at short distances and short times. To prove this look at a typical expectation value

$$\langle\mathcal{O}$$

$$^{n}\rangle_0 = \sum_\alpha \int_{k_i} f_\alpha(x_l - x_m, t_l - t_m, k_l, k_m) \times \mathrm{e}^{-\sum_{i,j} Q_{ij}k_ik_j}, \tag{14}$$

where each contribution consists of a function f_α, which is a product of propagators, correlators and k's and an exponential factor, with

$$Q_{ij} = -C(x_i - x_j, t_i - t_j). \tag{15}$$

f_α is a regular function of the distances. Divergences at finite distances can only occur if Q_{ij} is not a positive form. We will show that Q_{ij} is a positive form for all k_i which satisfy the constraint

$$\sum_i k_i = 0. \tag{16}$$

This constraint always holds, see equation (12). For equal times it is just the statement that the Coulomb energy of a globally neutral assembly of charges is positive. One simply identifies C with the Coulomb-propagator and k_i with the charges. In the dynamic case, write

$$Q_{ij} = (2-D)S_D\int\frac{\mathrm{d}^Dp}{(2\pi)^D}\int\frac{\mathrm{d}\omega}{2\pi}\frac{2\lambda}{\omega^2+(\lambda p^2)^2} \times\left(\mathrm{e}^{ip(x_i-x_j)+i\omega(t_i-t_j)}-1\right). \tag{17}$$

The exponential in (14) now is

$$\sum_{i,j} k_ik_jQ_{ij} = (2-D)S_D\int\frac{\mathrm{d}^Dp}{(2\pi)^D}\int\frac{\mathrm{d}\omega}{2\pi}\frac{2\lambda}{\omega^2+(\lambda p^2)^2} \times\sum_{i,j}k_ik_j\left(\mathrm{e}^{ip(x_i-x_j)+i\omega(t_i-t_j)}-1\right)$$
$$= (2-D)S_D\int\frac{\mathrm{d}^Dp}{(2\pi)^D}\int\frac{\mathrm{d}\omega}{2\pi}\frac{2\lambda}{\omega^2+(\lambda p^2)^2} \times\left|\sum_i k_i\,\mathrm{e}^{ipx_i+\omega t_i}\right|^2. \tag{18}$$

To get the second line, equation (16) has been used. Note that again due to equation (16), the integral is ultraviolet convergent and thus positive. It vanishes if and only if the charge-density, regarded as a function of space *and* time, vanishes. This is possible if and only if endpoints of the dipoles (which form the interaction) are at the same point in space *and* time. No divergence occurs at finite distances. To renormalize the theory, only short distance divergences have to be removed by adding appropriate counter-terms. In addition, as the divergences occur at short distances, they can be analyzed *via* a multilocal operator product expansion (MOPE) [11]. For a detailed discussion of the MOPE and examples see [11,24,13,12].

We show now that the counter-terms which render the static theory (1) finite are also sufficient for the dynamical case (4). As an illustration we first calculate the 1-loop counter-terms for the renormalization of the field and of the coupling-constant. The first singular configuration appears, when both ends of the interaction vertex (12) are contracted towards a single point. The leading term

(MOPE-coefficient) of this expansion is (the dotted line indicates points which are contracted):

$$= 2\int_k -[\tilde{r}(x,t)(ik)]\,\mathrm{e}^{-k^2|x-y|^{2-D}} \times[(ik)(r(x,t)-r(y,t))]+\ldots \quad (19)$$

We now Taylor-expand $r(y,t)$ as

$$r(y,t) = r(x,t) + (y-x)\nabla r(x,t) + \frac{1}{2}\left[(y-x)\nabla\right]^2 r(x,t) + \mathcal{O}(|x-y|^3). \quad (20)$$

The leading term in equation (19) is

$$\int_k \tilde{r}(x,t)(-\Delta r(x,t))\frac{(x-y)^2}{D}\frac{-k^2}{d}\mathrm{e}^{-k^2|x-y|^{2\nu}} = \tilde{r}(x,t)(-\Delta)r(x,t)\left(\frac{-1}{2D}\right)|x-y|^{D-\nu d}. \quad (21)$$

Denoting with $\left(\;\;\middle|\;\;\right)$ the MOPE coefficient of  proportional to  $= \tilde{r}(x,t)(-\Delta)r(x,t)$, this can be written in the form

$$\left(\;\;\middle|\;\;\right) = \left(\;\;\middle|\;\;\right) \quad (22)$$

where

$$\left(\;\;\middle|\;\;\right) = -\frac{1}{2D}|x-y|^{D-\nu d} \quad (23)$$

is the static MOPE-coefficient [12,13]. This implies that the counter-term for the wave-function renormalization is the same as in the static case. Let us now regard the counter-term for the coupling-constant renormalization. Using the techniques of [11–13], we obtain for the contraction of two interaction vertices to one interaction vertex:

$$\left(\;\;\middle|\;\;\right) = \frac{d}{4}(G(x,t)+G(y,t)) \times\left[C(x,t)+C(y,t)\right]^{-d/2-1} \quad (24)$$

x and y are the distances of the contracted endpoints of the dipoles, t is their time-difference. The trick is now to write this expression with the help of equation (11) as

$$-\frac{1}{2\lambda}\Theta(t>0)\frac{\mathrm{d}}{\mathrm{d}t}\left[C(x,t)+C(y,t)\right]^{-d/2}. \quad (25)$$

In performing the perturbation-theory, we have to integrate over all times. If we use, as is usually done, no cut-off in the time-direction, the time integral will simply give the value of the function at its lower bound:

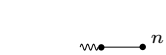
$$\int_0^\infty \mathrm{d}t\left(\;\;\middle|\;\;\right) = \frac{1}{2\lambda}\left(\;\;\middle|\;\;\right). \quad (26)$$

(The r.h.s. is the counter-term of the static theory, see [11–13].) We easily convince ourselves that this relation implies the same counterterm as in the static case, if we take care of the additional combinatorial factor 2 for the time-ordering of the interaction vertices.

One knows from general arguments that the divergences associated to short distances in space are removed by the static counter-terms [27]. We now use the fact that the static theory is renormalizable [11]. This implies that new divergences can only appear for short times. We therefore have to analyse all possible divergences of this type. Using the MOPE, the most relevant divergences are associated to the operator

 $= \tilde{r}(x,t)\dot{r}(x,t).$ (27)

We now regard a general contraction of n dipoles towards :

 (28)

In order to obtain the operator , one has to contract all fields r and $\tilde{r}$ except of the field $\tilde{r}(z,t)$ with the largest time-argument (all other contractions give 0). One also has to leave uncontracted one arbitrarily chosen field r. Due to the structure of the interaction (12), the field r always appears in the form $r(x,t-\tau)-r(y,t-\tau)$. So the contraction yields

$$\tilde{r}(z,t)\left[r(x,t-\tau)-r(y,t-\tau)\right]M(\text{distances}), \quad (29)$$

where M denotes the MOPE-coefficient which depends on the distances in space and time. Now, $r(x,t-\tau)-r(y,t-\tau)$ has to be expanded about (z,t). The leading term has at least *one* spatial gradient. No term of the form of (27) can be constructed. Therefore, there is no singular contribution of this type in any order in perturbation theory and no renormalization of is needed. The last at $\varepsilon = 0$ marginal operator is

 $= \tilde{r}(x,t)^2.$ (30)

Its renormalization is given by the fluctuation-dissipation theorem, see below.

We now introduce renormalized quantities according to

$$\begin{aligned} r &= \sqrt{Z}\,r_{\mathrm{R}} \\ \tilde{r} &= \sqrt{\tilde{Z}}\,\tilde{r}_{\mathrm{R}} \\ \lambda &= Z_\lambda \lambda_{\mathrm{R}} \\ b &= b_{\mathrm{R}} Z_b Z^{d/2}\mu^\varepsilon \end{aligned} \quad (31)$$

ε is the dimensional regularization parameter, defined by

$$\varepsilon = 2D - \nu d. \quad (32)$$

The fluctuation-dissipation theorem states that

$$\Theta(t)\frac{\partial}{\partial t}\left\langle \frac{1}{2}\left(r(x,t)-r(0,0)\right)^2\right\rangle = \lambda\left\langle r(x,t)\tilde{r}(0,0)\right\rangle. \quad (33)$$

This relation for the full expectation-values holds as well for renormalized as for bare quantities. We therefrom deduce that

$$Z_\lambda = \sqrt{\frac{Z}{\tilde{Z}}}. \quad (34)$$

In addition, our findings that the term ⟿ has not to be renormalized indicate that

$$\sqrt{Z\tilde{Z}} = 1. \tag{35}$$

It follows that

$$Z_\lambda = Z. \tag{36}$$

As usual, the renormalization group β-function $\beta(b_{\rm R})$ and the full scaling dimension $\nu(b_{\rm R})$ of r are obtained from the variation of the coupling constant and the field with respect to the renormalization scale μ, keeping the bare couplings fixed. They are written in terms of Z and Z_b as

$$\beta(b_{\rm R}) = \frac{-\varepsilon b_{\rm R}}{1 + b_{\rm R}\frac{\partial}{\partial b_{\rm R}}\ln Z_b + \frac{d}{2} b_{\rm R}\frac{\partial}{\partial b_{\rm R}}\ln Z} \tag{37}$$

$$\nu(b_{\rm R}) = \frac{2-D}{2} - \frac{1}{2}\beta(b_{\rm R})\frac{\partial}{\partial b_{\rm R}}\ln Z. \tag{38}$$

The β-function has an IR-fixed point for $b_{\rm R}^* > 0$. The function $\nu(b_{\rm R})$ is for large scales therefore given by

$$\nu^* = \nu(b^*). \tag{39}$$

We deduce that the correlation-function scales for equal times as

$$\left\langle (r(x,0) - r(0,0))^2 \right\rangle \sim |x|^{2\nu^*}. \tag{40}$$

If we define the exponent z for the auto-correlation-function as

$$\left\langle (r(0,t) - r(0,0))^2 \right\rangle \sim |t|^{2/z}, \tag{41}$$

the exponents z and ν^* are dependent:

$$z = 2 + \frac{D}{\nu^*}\cdot \tag{42}$$

For polymers ($D = 1$), this relation was given using scaling arguments in [14], for membranes ($D = 2$) in [16]. This result was followed by perturbative calculations for polymers in 1-loop [17–22] and 2-loop [23] order.

It is interesting to note that for polymers and membranes (42) can be written in the form

$$z = 2 + d_{\rm f} \tag{43}$$

where $d_{\rm f}$ is the fractal dimension of the membrane or the polymer.

It seems attractive to use (43) in Monte Carlo simulations to improve the determination of $d_{\rm f}$ for selfavoiding membranes. This might be advantageous as z can be measured for any point and any time, *i.e.* on a large statistical ensemble.

In conclusion: We have shown that the purely diffusive motion of polymers and polymerized membranes is given to all orders in perturbation theory by (43). Experimentally however, an additional hydrodynamic interaction is present. This question will be addressed in a subsequent publication [28].

It is a pleasure to thank F. David, H. W. Diehl and L. Schäfer for stimulating discussions.

References

1. J. des Cloizeaux, G. Jannink, *Polymers in Solution, Their Modelling and Structure*, (Clarendon Press, Oxford, 1990).
2. F.F. Abraham, W.E. Rudge, M. Plischke, Phys. Rev. Lett. **62**, 1757 (1989).
3. F.F. Abraham, D.R. Nelson, J. Phys. France **51**, 2653 (1990).
4. G.S. Grest, M. Murat, J. Phys. France **51**, 1415 (1990).
5. G.S. Grest, I.B. Petsche, Phys. Rev. **E 50**, 1737 (1994).
6. D.M. Kroll, G. Gompper, J. Phys. I France **3**, 1131 (1993).
7. T. Hwa, E. Kokufuta, T. Tanaka, Phys. Rev. **A 44**, 2235 (1991).
8. M.S. Spector, E. Naranjo, S. Chiruvolu, J.A. Zasadzinski, Phys. Rev. Lett. **73**, 2867–2870 (1994).
9. J.A. Aronowitz, T.C. Lubensky, Europhys. Lett. **4**, 395–401 (1987).
10. M. Kardar, D.R. Nelson, Phys. Rev. Lett. **58**, 1289 and 2280 E (1987).
11. F. David, B. Duplantier, E. Guitter, Phys. Rev. Lett. **72**, 311 (1994).
12. F. David, K.J. Wiese, Phys. Rev. Lett. **76**, 4564 (1996).
13. K.J. Wiese, F. David, Nucl. Phys. **B 487**, 529–632 (1997).
14. P.G. De Gennes, Macromolécules **9**, 587 (1976).
15. P.G. De Gennes, Macromolécules **9**, 594 (1976).
16. Y. Kantor, M. Kardar, D.R. Nelson, Phys. Rev. **A 35**, 3056–3071 (1987).
17. G.F. Al-Noaimi, G.C. Martinez-Mekler, C.A. Wilson, J. Phys. Lett. France **39**, 373 (1978).
18. Y. Oono, K.F. Freed, J. Chem. Phys. **75**, 1009 (1975).
19. Y. Oono, *Dynamics in polymer solutions: A renormalization group approach* (Wiley, New York, 1985).
20. S. Puri, B. Schaub, Y. Oono, Phys. Rev. **A 34**, 3362 (1986).
21. Shi-Qing Wang, K.F. Freed, J. Chem. Phys. **85**, 6210 (1986).
22. B. Schaub, D.B. Creamer, H. Johannesson, J. Phys. **A 21**, 1431–1455 (1988).
23. L. Schäfer, private communication.
24. F. David, B. Duplantier, E. Guitter, *Renormalization theory for the self-avoiding polymerized membranes*, cond-mat **9702136** (1997).
25. P.C. Hohenberg, B.I. Halperin, Rev. Mod. Phys. **49**, 435 (1977).
26. H.K. Janssen, *On the renormalized field theory of nonlinear crtitical relaxation*, in "*From Phase Transitions to Chaos*" of "*Topics in Modern Statistical Physics*" (World Scientific, 1992), pp. 68–117.
27. J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena* (Oxford University Press, 1989).
28. K.J. Wiese, Eur. Phys. J. B **1**, 273 (1998).